Lecture 14

In this lecture we continue our study of logic.

Exerises on matrices will go up on Blackboard tomorrow afternoon or evening.

<u>More on Logic</u>

Negation of a proposition is the statement that the proposition is false.

E.g. Let P be the proposition:

the moon is bigger than the sun.

The negation is:

not P : <u>it's not true that</u> the moon is bigger than the sun.

Equivalently, the moon is smaller than (or possibly of equal size to) the sun.

We use various symbols for "not":

not P

$\neg P$

$\sim P$

$\overline{P}$

Eg. Which of the following is the negation of "all camels have humps"?

(i) no camels have humps

(ii) some camels have humps

(iii) some camels don't have humps ✓

(iv) all camels don't have humps $\Leftrightarrow$ (i)

(v) some camels don't know whether or not they have humps

Soln

P: all camels have humps

$\neg P$: it's not true that all camels have humps

In formal logic, "some" means "at least one". So it includes the possibility of "all" as a special case.

<u>E.g.</u>

Some dogs are animals.

True. (All dogs are animals.
$\therefore$ some dogs are animals.)

<u>Note</u>

Every proposition has a <u>truth value</u> of either T (for "true") or F (for "false").

If P is true, $\neg P$ is false.

If P is false, $\neg P$ is true.

More about implications

Let $P \Rightarrow Q$ be an implication.

Its converse is $Q \Rightarrow P$.

Its contrapositive is $\sim Q \Rightarrow \sim P$.

Its inverse is $\sim P \Rightarrow \sim Q$.

Note

An implication may be true while its converse is false (and vice versa).

E.g. If $\underbrace{\text{Bruce is a rabbit}}_{P}$ then $\underbrace{\text{Bruce is an animal.}}_{Q}$

if P then Q

$$P \Rightarrow Q$$

Converse: $Q \Rightarrow P$

if Q then P

If Bruce is an animal then Bruce is a rabbit.

Here, $P \Rightarrow Q$ is true while $Q \Rightarrow P$ is false.

It can be shown that <u>every implication is logically equivalent to its contrapositive.</u>

$$P \Rightarrow Q \equiv \neg Q \Rightarrow \neg P$$

<u>E.g.</u>

If Bruce is a rabbit then Bruce is an animal.

Contrapositive:

If Bruce is not an animal then Bruce is not a rabbit.

These clearly have the same meaning.

Quantifiers

These are words or phrases like:

some
all
none
there exists
most
one

They get applied to variables to create more complicated statements.

E.g. Some integers are positive. [True]

All integers have real square roots. [False]

No rational numbers equal π. [True]

There exists a positive integer which is smaller than every other positive integer. [True]

$n = 1$ — the smallest positive integer.